Original Research

Comparison of Four Different Lactation Curve Models for Prediction of Fortnightly Test Day Milk Yields in First Lactation Rathi Cattle

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Abstract

Data of 3943 fortnightly test day milk yield (FTDMY) records during first lactation of 208 Rathi cattle calved during 1985-2014 at the Livestock Research Station, Nohar were used in the present investigation. The least squares means of the FTDMYs ranged from 3.85 kg to 8.4 kg. The relative efficiency of four lactation curve models viz. Exponential function, Parabolic function, Inverse Polynomial function and Gamma function were compared. Inverse polynomial function described the highest coefficient of determination (R^2 = 99.92%) and with least value (0.0604 kg) of root mean squares error (RMSE), suggesting inverse polynomial function being best mathematical model for prediction of fortnightly test day milk yields in Rathi Cattle.

Key words: Exponential Function, FTDMYs, Gamma-Type Function, Inverse Polynomial Function, Lactation Curve, Parabolic Function, Rathi

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Introduction

Lactation curve is the graphical representation of milk yield against time during lactation in dairy animals (Brody *et al.*, 1923). The shape of lactation curve provides valuable input for taking appropriate decisions on culling, health, breeding and feeding management of dairy animals. The main aim of lactation curve modelling is to predict milk yield at any point of lactation with minimum error in case of inconsistent milk recordings. A mathematical function can represent the lactation curve if the accuracy of fitting the function is very good (Singh *et al.*, 2017). Several mathematical models (linear and non-linear) have been developed to describe the shape of lactation curves in bovine and bubalus species in the past. But, the model of best fit is still elusive, because of the influence of various environmental factors (Sahoo *et al.*, 2015).

Materials and Methods

Data of 3943 fortnightly test day milk yield (FTDMY) records during first lactation of 208 Rathi cattle calved during 1985-2014 at the Livestock Research Station, Nohar were collected. A total of 21 fortnightly test day milk yield records (6th day, 20th day, 35th day, 50^{th} day... 305th day) were taken at an interval of 15 days from daily milk yield recording register.

Statistical Methodology

The least squares means of the FTDMYs were calculated for adjustment of effect of non-genetic factors (season and year of calving). The various mathematical models describing the shape of lactation curve given by Brody *et al*. (1923), Sikka (1950), Nelder (1966) and Wood (1967) were utilized in the present investigation to find out best fit curve. The functions were analyzed by multiple regression analysis to obtain the model parameters (A, b, c, b_0, b_1 and b_2)-

1. Exponential function (Brody *et al*., 1923)

$$Y_t = A\ e^{-ct}$$

Exponential function by using logarithmic form- $Y_t = \ln Y = \ln A - ct$

2. Parabolic function (Sikka, 1950)

$$Y_t = A e^{bt + ct^2}$$

Parabolic function by using base and linear regression model- $Y_t = \ln Y = \ln A + bt + ct^2$

3. Inverse Polynomial function (Nelder, 1966)

$$Y_t = t(b_0 + b_1 t + b_2 t^2)^{-1}$$

After dividing by t on both sides and taking reciprocals it can be written as-

$$t/Y_t = b_0 + b_1 t + b_2 t^2$$

4. Gamma type function (Wood, 1967)

$$Y_t = A\ t^b\ e^{-ct}$$

Gamma function in its log base and linear regression model- $Y_t = \ln Y = \ln A + b \ln t - ct$

Where,

Y_t = average daily yield at t_{th} test day of lactation

A = initial milk yield after calving

b = inclining slope parameter up to the peak yield

c = descending slope parameter (persistency measure)

b_0= theoretical value at the time of parturition

b_1= rising extremes of the curve

b_2= declining extremes of the curve

t = length of time since calving

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The efficacy of the above functions was compared by the coefficient of determination (R^2) and the Root mean square error (RMSE).

1. The coefficient of determination was calculated as-

$$R^2 = (\sum_i (Y_i - \bar{Y})^2 \sum_i (Y_i - \hat{Y}_i)^2 / \sum_i (Y_i - \bar{Y})^2)$$

Where,
Y = observed value
$\bar{Y}$ = mean of observed values
$\hat{Y}_i$= estimated value

2. The Root mean square error which is a kind of generalized standard deviation-

$$RMSE = \frac{[\sum_{i=1}^{n}(Y_i - \hat{Y})^2]^{0.5}}{n}$$

Where,
n= number of observations
Y_i = actual values
$\hat{Y}$ = values predicted by the regression model

Smaller RMSE indicates better modelling.

Results and Discussions

The least squares means of the FTDMYs were calculated for adjustment of effect of non-genetic factors (season, year of calving and age at first calving groups). The initial milk yield in Rathi cattle was observed as 5.93 kg (6^{th} day). The peak milk yield of 8.4 kg approached in the third week i.e. second test day (20^{th} day) and subsequently declined to 3.85 kg in 21^{st} fortnight (305^{th} day). Similar findings were reported by Yadav (1981) in Hariana cattle and Dhaka (2013) in Rathi cattle. Lactation curve parameters (A, b, c, b_0, b_1 and b_2) were estimated by multiple regression analysis to fit into the mathematical models to obtain the prediction equation for FTDMYs (Table 2). The prediction equations for these mathematical functions are given in Table 1.

Table 1: Different lactation curve functions with their prediction equations, R^2 and RMSE value in Rathi cattle

S. No.	Lactation Curve Functions	Equations	R^2(%)	RMSE(Kg)
1	Exponential	$Y_t = (8.3963).e^{-(0.03690)t}$	88.45	0.125
2	Parabolic	$Y_t = (7.9988).e^{\{(-0.02424)t+(-0.00058)t^2\}}$	89.08	0.1205
3	Inverse Polynomial	$Y_t = t.(0.06496 + 0.08157.t + 0.008269.t^2)^{-1}$	99.92	0.0604
4	Gamma	$Y_t = (7.3769).t^{(0.15919)}.e^{-(0.05641)t}$	92.31	0.0989

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Table 2: Estimated lactation curve parameters of different functions for prediction of FTDMYs

S. No.	Lactation Curve Functions	Equations	Parameters					
			A	B	C	b0	b1	b2
1	Exponential	$Y_t = A.e^{-ct}$	8.3963	-	0.0369	-	-	-
2	Parabolic	$Y_t = A.e^{bt+ct^2}$	7.9988	-0.02424	-0.00058	-	-	-
3	Inverse Polynomial	$Y_t = t.(b_0 + b_1.t + b_2.t^2)^{-1}$	-	-	-	0.06496	0.08157	0.008269
4	Gamma	$Y_t = A.t^b.e^{-ct}$	7.3769	0.15919	0.05641	-	-	-

The coefficient of determination (R^2) and the root mean square error value (RMSE) for the four functions is given in Table 1 and the curves of predicted milk yield by these four functions along with the actual milk yield is presented in Figs. 1-5. Maximum R^2 value was observed for inverse polynomial function (R^2 = 99.92%), followed by Gamma function (R^2 = 92.31%), Parabolic function (R^2 = 89.08%) and exponential function (R^2 = 88.45%).

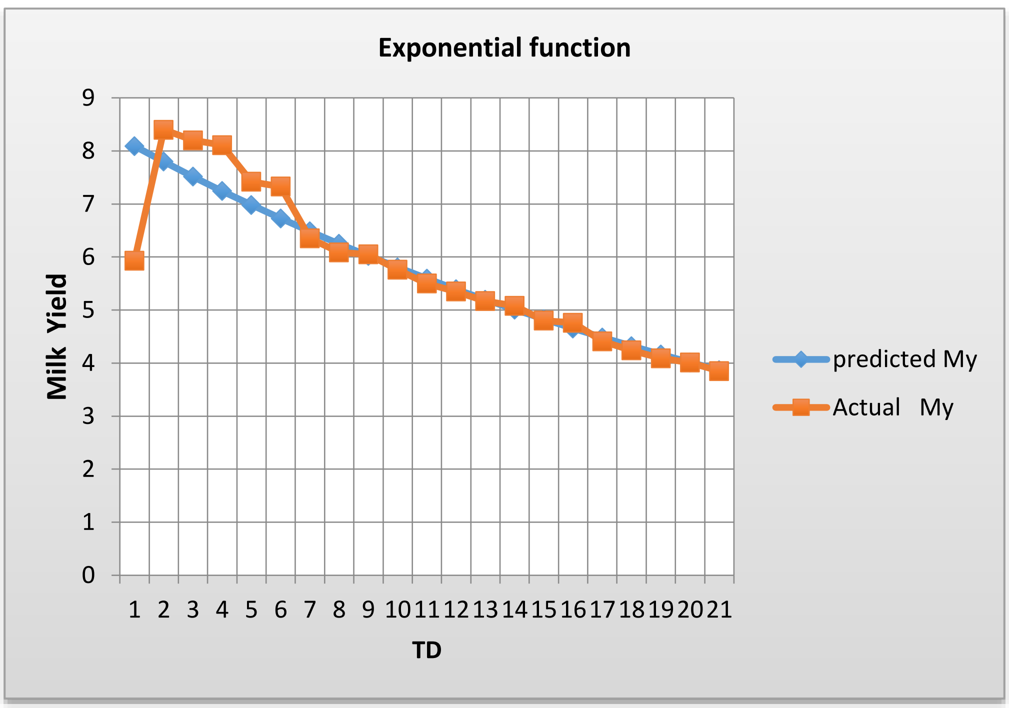

Fig. 1: Observed and predicted FTDMYs for first lactation by exponential function

The root mean square error (RMSE) was least for inverse polynomial function (0.0604 kg), followed by gamma function (0.0989 kg), parabolic function (0.1205 kg) and maximum for exponential function (0.125 kg) confirming the same order of superiority. The same order of superiority of these functions was observed by Yadav (1981) in Hariana cattle, Gahlot (1986) in Rathi cattle, Gandhi and Dongre (2013) in Sahiwal cattle and Singh *et al*. (2017) in Murrah buffaloes.

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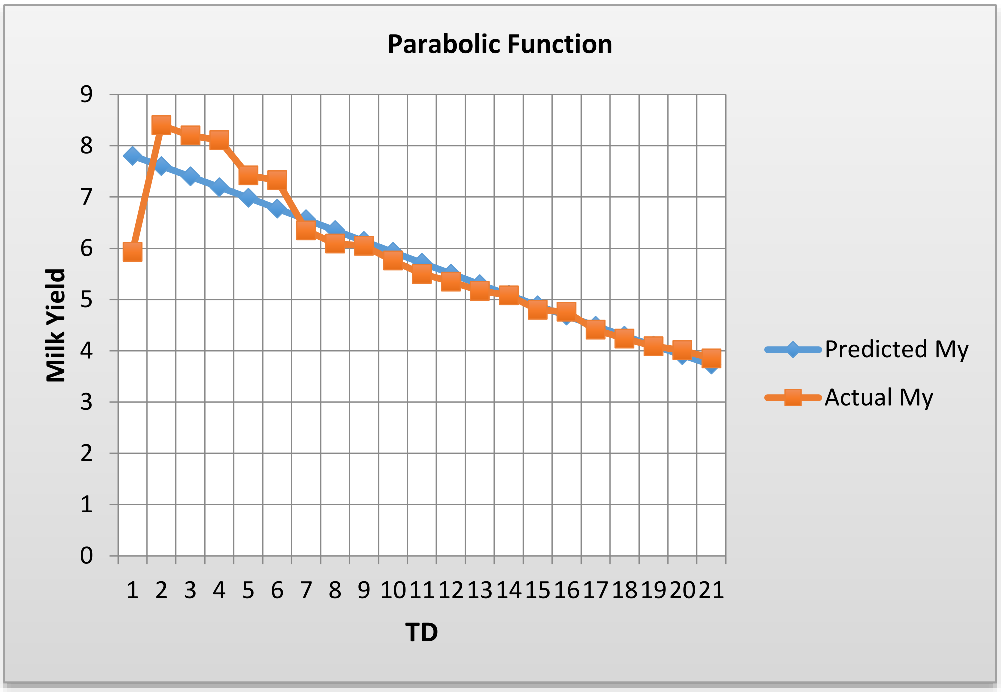

Fig. 2: Observed and predicted FTDMYs for first lactation by parabolic function

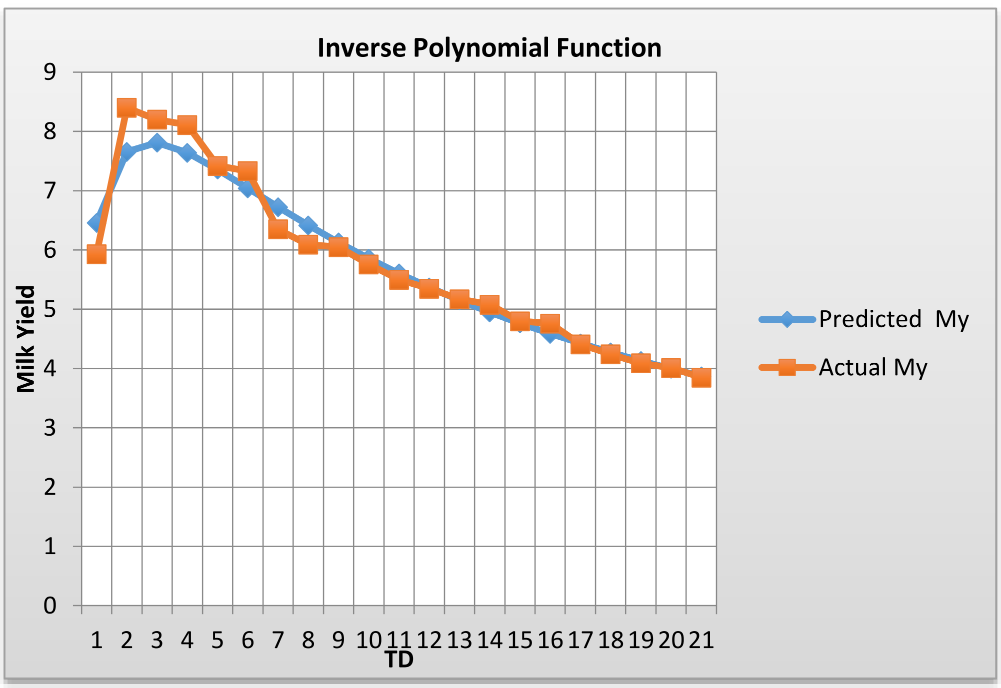

Fig. 3: Observed and predicted FTDMYs for first lactation by inverse polynomial function

The lactation curves of the four functions i.e. exponential, parabolic, inverse polynomial and gamma function fitted for fortnightly test day milk yield in Rathi cattle are presented in Figs. 1-4.

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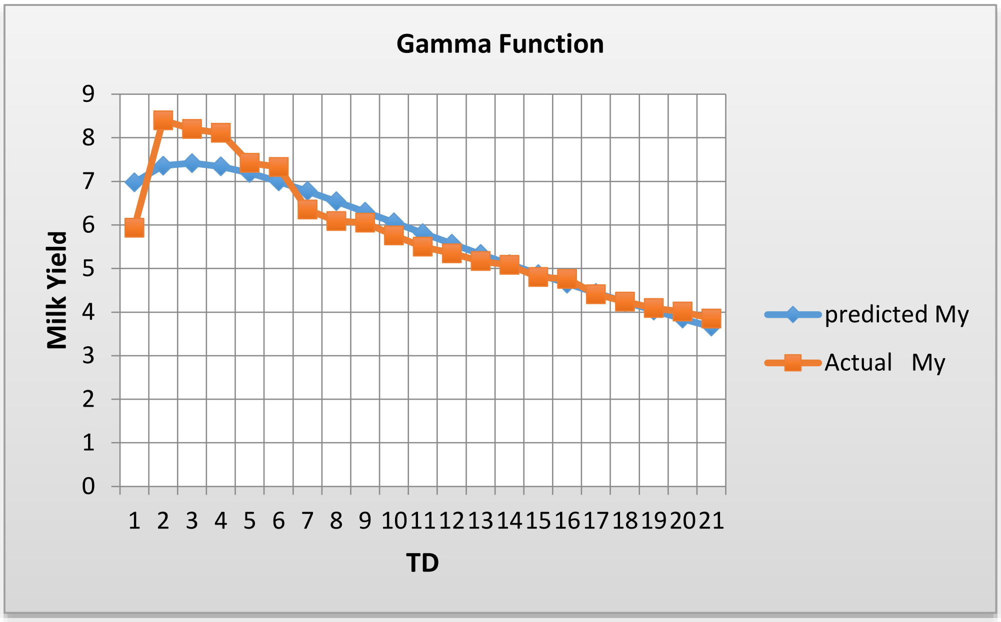

Fig. 4: Observed and predicted FTDMYs for first lactation by gamma function

Table 3: Predicted FTDMY (Kg) and Error (Kg) of different lactation curve functions in Rathi cattle

FTDMY	**Observed Value** {LS means (kg)}	**Exponential Function**		**Parabolic Function**		**Inverse Polynomial Function**		**Gamma Function**	
		Predicted Value(kg)	**Error (kg)**	**Predicted Value (kg)**	**Error (kg)**	**Predicted Value(kg)**	**Error (kg)**	**Predicted Value(kg)**	**Error (kg)**
TD1	5.93	8.09	2.16	7.8	1.87	6.46	0.53	6.97	1.04
TD2	8.4	7.8	-0.6	7.6	-0.8	7.66	-0.74	7.36	-1.04
TD3	8.2	7.52	-0.68	7.4	-0.8	7.81	-0.39	7.42	-0.78
TD4	8.11	7.24	-0.87	7.19	-0.92	7.64	-0.47	7.34	-0.77
TD5	7.42	6.98	-0.44	6.98	-0.44	7.36	-0.06	7.19	-0.23
TD6	7.33	6.73	-0.6	6.77	-0.56	7.04	-0.29	6.99	-0.34
TD7	6.35	6.49	0.14	6.56	0.21	6.72	0.37	6.78	0.43
TD8	6.09	6.25	0.16	6.35	0.26	6.42	0.33	6.54	0.45
TD9	6.05	6.02	-0.03	6.14	0.09	6.13	0.08	6.3	0.25
TD10	5.76	5.81	0.05	5.93	0.17	5.86	0.1	6.05	0.29
TD11	5.5	5.6	0.1	5.71	0.21	5.6	0.1	5.81	0.31
TD12	5.35	5.39	0.04	5.5	0.15	5.37	0.02	5.57	0.22
TD13	5.17	5.2	0.03	5.3	0.13	5.15	-0.02	5.33	0.16
TD14	5.08	5.01	-0.07	5.09	0.01	4.95	-0.13	5.1	0.02
TD15	4.8	4.83	0.03	4.89	0.09	4.76	-0.04	4.87	0.07
TD16	4.76	4.65	-0.11	4.68	-0.08	4.59	-0.17	4.65	-0.11
TD17	4.41	4.48	0.07	4.49	0.08	4.43	0.02	4.44	0.03
TD18	4.24	4.32	0.08	4.29	0.05	4.27	0.03	4.23	-0.01
TD19	4.09	4.17	0.08	4.1	0.01	4.13	0.04	4.04	-0.05
TD20	4.01	4.01	0	3.91	-0.1	4	-0.01	3.85	-0.16
TD21	3.85	3.87	0.02	3.73	-0.12	3.87	0.02	3.66	-0.19

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These curves were constructed on the basis of "Predicted FTDMY (Kg)" and "Error (Kg)" of different lactation curve functions in Rathi cattle as given in Table 3**.** Graphical comparison of the above models showed that exponential and parabolic functions explained only the declining trend and took no account of the ascending phase. While inverse polynomial and gamma type functions estimated an initial ascending phase followed by peak and descending phase with the advancement of lactation. Inverse polynomial function was more close to the observed lactation curve. Similar results were reported by Savalia *et al.* (2017) in Gir cattle. The 3-parameter based incomplete gamma function of Wood (1967) is one of the widely used function to depict various types of lactation curves in dairy animals, as the model had the advantage of having a limited number of parameters with an assessed direct reference to main features of lactation curve (Rekaya *et al.*, 2000).

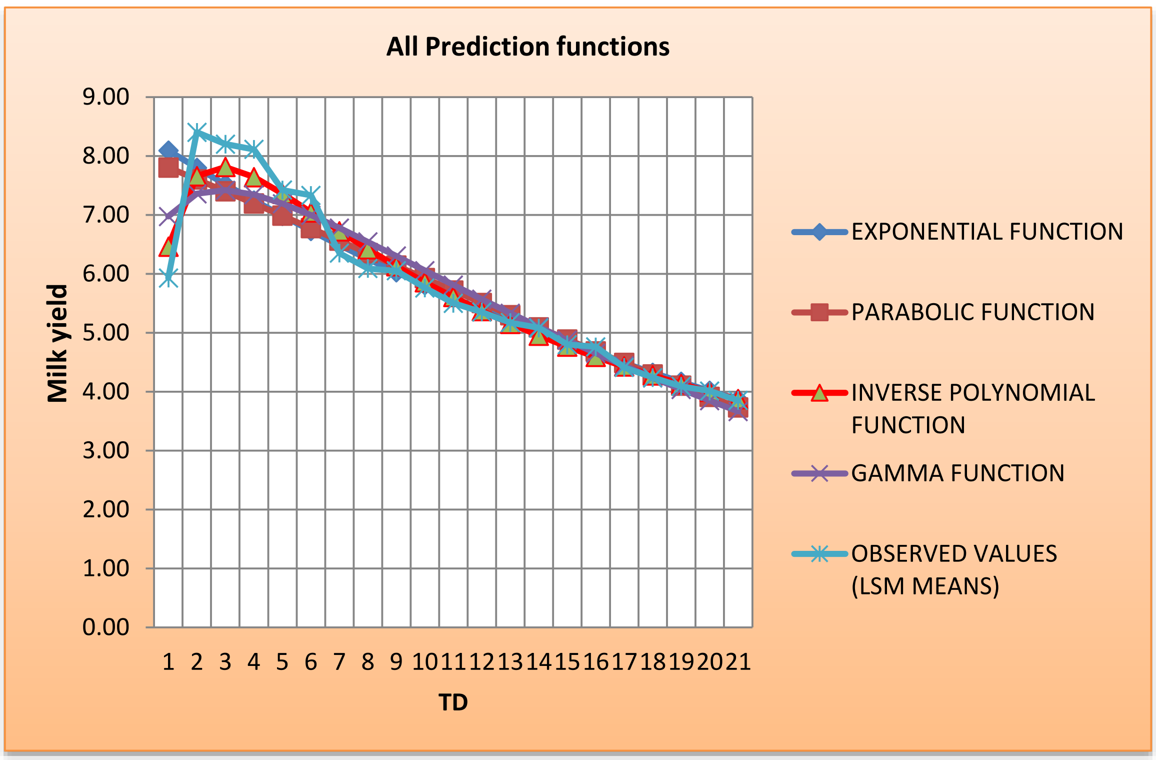

Fig. 5: Observed and predicted FTDMYs by various lactation curve functions in Rathi cattle

In the present study, gamma function was found to be the second best fit function (high R^2=92.31% and low RMSE=0.0989 kg) for prediction of FTDMYs in Rathi cattle (Table 1). Vohra *et al.* (2011) utilized adjusted FTDMYs of first three lactations for estimating the wood function parameters in Karan Fries crossbred cows and reported that the gamma function was better fit for first lactation records in Karan Fries cows. Jingar *et al.* (2014) analysed the lactation curve of normal and mastitis Karan Fries cows using gamma type function on fortnightly test day milk yield records. The higher coefficient of determination

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(R^2) and lower root mean square error indicated the goodness and accuracy of the model for prediction of milk production in field conditions. Similar results were reported by Kshandakar *et al.* (2017) in Vrindavani crossbred cattle. Dongre and Gandhi (2014) utilized weekly test day milk yield records of Sahiwal cows to compare three lactation curve models *viz.* quadratic model, gamma type function and mixed log function. Gamma function gave high R^2 (95.6%) with low RMSE (0.037 kg). It was recommended that gamma function can give best fit for the low yielder cows. Inverse polynomial function described the highest coefficient of determination (R^2 = 99.92%) and with least value (0.0604 kg) of root mean squares error (RMSE). Hence, it can be concluded that inverse polynomial function is the best mathematical model for prediction of fortnightly test day milk yields in Rathi cattle. These results are in close accordance with Yadav (1981) in Hariana cattle, Gahlot (1986) in Rathi cattle, Gandhi and Dongre (2013) in Sahiwal cattle and Singh *et al*. (2017) in Murrah buffaloes.

Conclusion

In the present investigation it was found that on an average, the peak yield from all lactation curve functions was found to be highest around 20th day of lactation in Rathi cattle. Inverse polynomial function is the best function among four models studied, followed by Gamma function explaining highest coefficient of determination with lowest root mean square error and therefore these two functions are recommended for prediction of FTDMYs in Rathi cattle.

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