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Modelling Lactation Performance in Frieswal Cows: A Comparative Study of Mathematical Models

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Abstract

The present study was undertaken to compare four non-linear lactation curve models (Gamma-Type (GT), Mixed Logarithmic (ML), Wilmink (WL) and Quadratic (QD)) to describe the monthly milk yield patterns in Frieswal cows using the data of five lactations (2019-2024) at JNKVV, Jabalpur. Model performance was evaluated on criteria of peak yield, persistency, time to peak production and goodness of fit. The Gamma-Type model had the best overall accuracy, particularly at the end of lactation, with high and consistent coefficients of determination (R2adj = 0.977 – 0.988). The best fit for cows in their first lactation was obtained by the Mixed Logarithmic model (R2adj = 0.943), indicating that this model is appropriate for early parity dynamics. The Wilmink model matched the early lactation trends well, but with a relatively lower explanatory power (R2adj = 0.921 – 0.977). The Quadratic model showed little flexibility to describe the non-linear nature of lactation curves (R2adj = 0.918 – 0.978). Peak milk yield of Frieswal cows was observed in the second month of lactation. The findings highlight the significance of parity-based model selection to enhance the accuracy of milk yield prediction and enable more effective herd management strategies.

Keywords: Gamma-Type Function, Frieswal Cows, Lactation Curve, Non-Linear Models, Peak Yield, Persistency of Production.

Introduction

The characterization of lactation curves represents a cornerstone of dairy cattle science, offering essential insights into the biological mechanisms underlying milk production, peak yield, and persistency. As highlighted by Arreola *et al*. (2004), modeling lactation dynamics is vital for accurately predicting total milk yield, enhancing herd management strategies, and guiding genetic selection aimed at improving dairy productivity. Madalena *et al*. (1979) further emphasized that milk production typically follows a well-defined trajectory: a sharp rise immediately after calving, attainment of peak yield during early to mid-lactation, and a gradual decline thereafter. Consequently, Macciotta *et al*. (2011) highlighted the importance of selecting appropriate mathematical models to capture breed-specific production characteristics and obtain robust estimates of key lactation parameters.

Mohanty *et al*. (2017) evaluated five models namely Sikka (SIK), Wilmink (WK), Morant and Gnanasakthy (MG), Ali and Schaeffer (AS) and Mitscherlich Exponential (ME) for a 150-day lactation period in crossbred dairy cows. They used goodness-of-fit indicators including adjusted coefficient of determination (R^2 adj), root mean square error (RMSE), Akaike's information criterion (AIC), and corrected AIC and found that the Ali and Schaeffer model had the highest predictive accuracy (R^2adj = 97.8%) with the smallest RMSE whereas the Sikka model yielded the largest persistency estimates. The findings emphasize the need for model selection that reflects the biological pattern of milk production. Odegible *et al*. (2020) investigated the lactation characteristics of White Fulani and Sokoto Gudali cows, estimating parameters such as initial yield (a), rate of increase (b), and rate of decline (c) using Wood's gamma function and the Wilmink model. Their results suggest that positive parameter estimates generally correspond to normal lactation patterns, while negative estimates could be a sign of atypical lactation curves. Importantly, these models were found to support genetic improvement programs by accurately quantifying lactation characteristics. Raja(2020) reported low heritability estimates for individual test-day records as well as for cumulative 305-day milk yields, indicating low additive genetic variance among the sires used in the breeding program. However, high genetic correlations of early test day yields (TD135, TD165, TD45) and cumulative yields indicated that early lactation measurements can be used as effective selection criteria for total milk production.

The present investigation was conducted to assess and compare the performance of various non-linear lactation curve models in Frieswal cows. The main objectives were to (i) to elect the model that most accurately represents the lactation curve for Frieswal cattle using goodness-of-fit measures and (ii) to estimate key lactation parameters for accurate prediction of milk yield and effective genetic improvement strategies.

Material and Methods

Study Overview - The study was carried out at the Jawaharlal Nehru Krishi Vishwa Vidyalaya Dairy Farm, Jabalpur, The study utilized test-day milk yield records collected over an eight-year period (2018–2024) from 47 Frieswal cows. Milk yield was recorded twice daily, and test-day records were used for analysis. Statistical analysis was done by using R software.

Data Processing - Only complete and continuous lactation records were used for analysis to assure consistency of data. The data set consisted of test-day milk yield records for a period of eight years (2018-2024) of 47 Frieswal cows. The data were examined for completeness and records with missing or inconsistent observations were omitted. To facilitate the fitting of non-linear lactation curve models, monthly average milk yields for the first 10 months of each lactation were computed from test-day records. The processed dataset was formatted into a standardized format prior to fitting a model and statistical analysis.

Lactation Curve Models - To describe the lactation patterns of Frieswal cows, various non-linear models were applied to the average daily milk yield data.

Wood Model (Gamma-Type Function) The Wood model (Wood, 1967) is one of the most widely used models for describing lactation curves:

$$Y_t = at^b e^{-ct}$$

where, Y_t= milk yield at time t,a = initial milk yield,b=ascending slope parameter,c = descending slope parameter and e = the base of the natural logarithm.

Quadratic ModelQuinn (2005) provided a simple polynomial model useful for capturing the general lactation pattern

$$Y_t = a + bt - ct^2$$

where a, b, and c are parameters representing intercept, linear, and quadratic effects, respectively.

Mixed Log ModelGuo and Swalve (1995) introduced a flexible model capable of describing non-linear growth and tapering trends

$$Y_t = a + bt^{0.5} - c\ logt$$

WhereY_tThe dependent variable at time t, such as production output, sales, or any growth measure.

Wilmink Model: Known for effectively describing persistency in milk production (Wilmink,1987)

$$Y_t = a + be^{-kt} + ct$$

WhereY_tMilk yield at time t.

Gamma-type function: Similar to the Wood model, this function captures the initial rise, peak, and decline of lactation:

$$Y_t = at^b e^{-ct}$$

Model Evaluation Criteria: The models were evaluated using the following statistical criteria to determine the best fit:

$$\mathrm{R}^2_{\mathrm{adj}} = 1 - \left[\frac{(\mathrm{n}-1)}{(\mathrm{n}-\mathrm{p})}\right](1-\mathrm{R}^2)\text{and RMSE} = \frac{\sqrt{\mathrm{RSS}}}{\mathrm{n}-\mathrm{p}-1};$$

$$\mathrm{AIC} = \mathrm{n}\ln(\mathrm{RSS}) + 2\mathrm{p}\ \text{ and }\ \mathrm{BIC} = \mathrm{n}\ln\left(\frac{\mathrm{RSS}}{\mathrm{n}}\right) + \mathrm{p}\ln(\mathrm{n})$$

where $\boldsymbol{R^2 = 1 - \left(\frac{RSS}{TSS}\right)}$, RSS is the residual sum of squares, TSS is the total sum of squares, n is the total number of observations, and p represents the number of model parameters.

Four non-linear lactation curve models, namely Gamma-Type (GT), Mixed Logarithmic (ML), Wilmink (WL), and Quadratic (QD), were fitted to the test-day milk yield data to characterize the lactation patterns of Frieswal cows. Model performance was evaluated using adjusted coefficient of determination (R^2adj), Root Mean Square Error (RMSE), Akaike's Information Criterion (AIC), and Bayesian Information Criterion (BIC). Peak yield, month at peak yield, and lactation persistency were estimated using the Gamma-Type model following the methodology of Tekerli *et al*. (2000).

Results and Discussion

The findings are based on a comparative evaluation of all four non-linear lactation curve models. The results focus on key aspects of lactation dynamics, including peak yield, persistency, and the timing of peak production across different lactation stages.Table1 presents the monthly average milk yield (liters/day) across the first five lactations of Frieswal cows. The analysis showed that peak milk yield consistently occurred in the second month of lactation across all lactation orders in Frieswal cows. The peak yields were 12.7, 13.1, 13.1 12.8, and 12.7 liters/day for the first through fifth lactations, respectively.

The results are in agreement with the observations of Jingar *et al*. (2014) who reported maximum milk yield in fourth lactation of Karan Fries cows. In contrast, the incomplete Gamma function was successfully used by Rekik *et al*. (2003) for 8,640 lactation records of Holstein–Friesian cows from four herd types in Tunisia and they showed that it is appropriate for modelling lactation curves under different management conditions. Holstein–Friesian and Ayrshire cows had better lactation performance such as higher peak milk yield and greater lactation persistency than cows in typical smallholder dairy farms under improved husbandry conditions (Shija, 2022).

Table 1: Monthly average milk yield (liters/day) during the first five lactations of Frieswal cows

Month	First	Second	Third	Fourth	Fifth
1	11.4	12.4	11.8	12.6	13.1
2	12.7	13.1	13.1	12.8	12.7
3	11.6	11.3	11.9	12.3	12.4
4	11.8	11.1	12.2	12.2	11.9
5	10.1	9.91	10.6	10.3	10.4
6	10.2	9.16	9.77	9.74	9.33
7	9.61	8.32	8.44	8.82	8.85
8	7.73	7.96	7.75	7.83	7.86
9	5.93	6.76	6.81	6.83	6.86
10	5.74	6.12	6.61	6.42	6.53

The results of the non-linear modelling of the test-day milk yield (litres/day) for the first five lactations of Frieswal cows are given in Table 2. There were differences in the ability of the tested models to accurately describe the lactation curve. The model performance was compared using goodness-of-fit statistics including adjusted R 2, RMSE, AIC, and BIC, and results showed that some models fitted better according to the lactation order. The non-linear modelling adequately described the lactation curve in primiparous Frieswal cows and the mixed log function was found to best describe the lactation curve with the highest adjusted R2 and lowest RMSE, AIC and BIC values. The Wilmink model was the second best and the quadratic model was the worst model.

The goodness-of-fit statistics of the four nonlinear lactation curve models across different lactations are presented in Table 2, and observed and predicted lactation curves are presented in Figures 1-5. Models showed different performances according to lactation order, indicating that none of the models was equally efficient in all parity groups.

The Mixed Logarithmic (ML) model was the best fit in the first lactation with the highest adjusted R^2 and lowest RMSE, AIC, and BIC values among the other tested models. Thus, the ML model was better in capturing the gradual increase in the milk yield and the subsequent decline in primiparous Frieswal cows. Similarly, Mohanty *et al.* (2017) reported that the relatively smooth lactation pattern of first-lactation dairy cows. Conversely, for the second to fifth lactations, the Gamma-Type (GT) model was consistently superior to the other models. The best predictive accuracy and model efficiency was indicated by the highest adjusted R^2 and lowest RMSE, AIC, and BIC values. The findings reveal that the Gamma-type function accurately represents the biological pattern of milk production in multiparous Frieswal cows, characterized by a swift rise following calving, attainment of peak yield, and subsequent gradual decline. Similar results were discovered by Tekerli *et al*. (2000) and Rekik *et al*. (2003), who successfully applied Gamma-type functions to describe lactation curves in Holstein–Friesian cows and stated their applicability under different herd management systems.The rest of the models, the Wilmink (WL) model was usually the second best after the Gamma-Type model in later lactations, with satisfactory goodness-of-fit statistics and a favourable description of the overall lactation trend. The quadratic (QD) model had the lowest adjusted R^2 and the highest RMSE, AIC, and BIC values, suggesting poor predictive performance and limited flexibility to describe the non-linear nature of lactation curves. Similar limitations of polynomial models have been reported by Macciotta *et al*. (2011), who pointed out that simple polynomial functions often do not adequately describe the biological nature of dairy lactation. The differences in model performance between lactations result from differences in physiological maturity and milk yield dynamics. Primiparous cows tend to take longer to reach peak milk yield and have lower production than multiparous cows. Mature cows have a more prominent lactation curve and higher peak production. Therefore, the mixed logarithmic model appears to be more suitable for first-lactation animals, and the gamma-type model is more realistic for mature lactation curves.

Table 3 presents the estimated peak milk yield, lactation persistency and month of peak milk production for each model across lactations. Large differences between models and lactation orders indicated differences in the ability to describe different phases of the lactation curve. The mixed logarithmic model predicted the highest peak milk yield (27.05–30.18 L/day^{-1}) and demonstrated the model's ability to capture the rapid increase in milk production during early lactation. Conversely, the Gamma-Type model estimated moderate peak yields (16.10–20.00 L/day−1) with one lactation having a higher peak (29.30 L/day−1) and yielded a more biologically balanced estimate of milk production. The Wilmink model showed a less steep lactation curve but a lower peak yield (9.40–15.00 L/day−1). Intermediate estimates (10.20–12.70 L/day -1) were obtained with the quadratic model.

Table 2: Non-linear modelling of test-day milk yield (liters/day) for the first five lactations of Frieswal cows' (GT= Gamma-type function, ML= Mixed Log Model,WL= Wilmink Model,QD= Quadratic Model)

LO	Model	Parameters estimates (Standard error)			Goodness of fit			
		A	B	c	R^2adj	RMSE	AIC	BIC
1	GT	13.44(0.58)	0.39(0.09)	0.17(0.25)	0.932	0.53	23.8	25
	ML	23.38(1.41)	-12.03(1.66)	-8.82(1.60)	0.943	0.49	22	23.3
	WL	110.41(27.44)	-98.80(28.07)	-4.53(1.08)	0.937	0.51	0.94	24.3
	QD	11.77(0.74)	0.27(0.31)	0.09(0.03)	0.932	0.53	0.93	25
2	GT	14.20(0.32)	0.16(0.50)	0.12(0.01)	0.977	0.29	12.2	13.4
	ML	20.14(0.87)	-7.52(1.02)	4.22(0.99)	0.977	0.3	12.4	13.6
	WL	26.06(19.28)	-12.50(19.72)	-1.24(0.76)	0.966	0.36	16.1	17.3
	QD	13.59(0.51)	-0.64(0.21)	0.11(0.02)	0.966	0.36	16.1	17.3
3	GT	13.92(0.35)	0.3177(0.05)	0.15(0.01)	0.978	0.3	13.8	15
	ML	22.48(1.05)	-10.50(1.24)	7.29(1.20)	0.966	0.36	16.2	17.5
	WL	26.06(19.28)	-12.50(19.72)	-1.24(0.76)	0.921	0.56	24.7	25.9
	QD	12.95(0.80)	-0.22(0.33)	0.05(0.03)	0.918	0.57	25.1	26.3
4	GT	14.44(0.26)	0.25(0.04)	0.14(0.01)	0.986	0.24	8.14	9.35
	ML	22.10(0.76)	-9.51(0.90)	6.12(0.87)	0.983	0.26	9.79	11
	WL	54.78(20.32)	-41.37(20.78)	-2.37(0.80)	0.965	0.38	17.1	18.3
	QD	13.50(0.55)	-0.38(0.23)	0.04(0.02)	0.963	0.39	17.6	18.8
5	GT	14.75(0.24)	0.18(0.36)	0.13(0.01)	0.988	0.22	5.98	7.19
	ML	21.24(0.69)	-8.17(0.81)	4.71(0.79)	0.986	0.24	7.81	9.02
	WL	33.39(16.46)	-19.40(16.83)	-1.55(0.64)	0.977	0.31	12.9	14.1
	QD	14.05(0.44)	-0.62(0.18)	0.02(0.02)	0.978	0.31	13.1	14.4

Similar differences between the lactation models were reported by Rekik *et al*. (2003) and Tekerli *et al*. (2000). The lactation persistency also varied among the models. The mixed logarithmic model had the highest persistency values (3.30-3.75) suggesting its greater ability to describe the sustained milk production after the peak yield. The Gamma-type model showed moderate persistency (2.21–3.41) and the Wilmink model showed relatively lower values (2.15–2.60) referring to a slower decline in milk yield. The quadratic model had the highest variation (1.18–3.16) indicating the inconsistent performance over lactations. These observations indicate that the mixed logarithmic model is a good description of persistent lactation and the gamma type model is a good description of both peak production and decline thereafter. Models also differed in the estimated month of maximum milk production. The peak production mixed logarithmic model predicted peak production to be between 5.21 and 6.53 months. This suggests a long ascending phase. Gamma type model predicted peak yield was 4.03 and 5.67 months which were nearer to biological lactation pattern of Frieswal cows. In contrast, the Wilmink and Quadratic models estimated much earlier peaks (1.07-2.76 and 0.12-2.71 months, respectively) indicating that these models are more suitable to describe early lactation changes than the whole lactation cycle.

The results revealed that the mixed logarithmic model was the best for modeling first-lactation Frieswal cows, and the gamma-type model gave the most reliable and biologically plausible description of lactation performance from the second to the fifth lactation. The better performance of the Gamma-type model with realistic estimates of peak yield, persistency, and time to peak underlines the appropriateness of the model for genetic evaluation, production forecasting, and dairy herd management in Frieswal cattle.

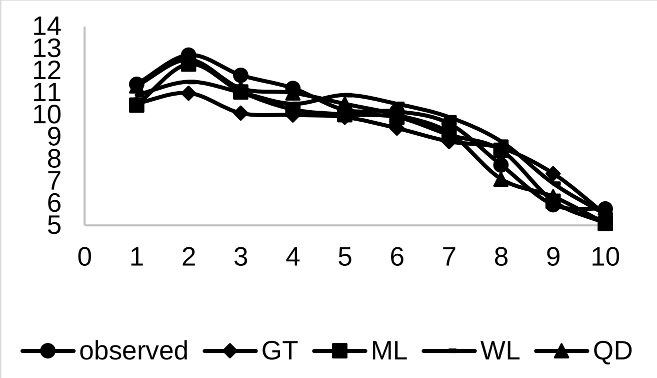

Fig 1: First lactation milk yield of Frieswal cows as estimated by non-linear models

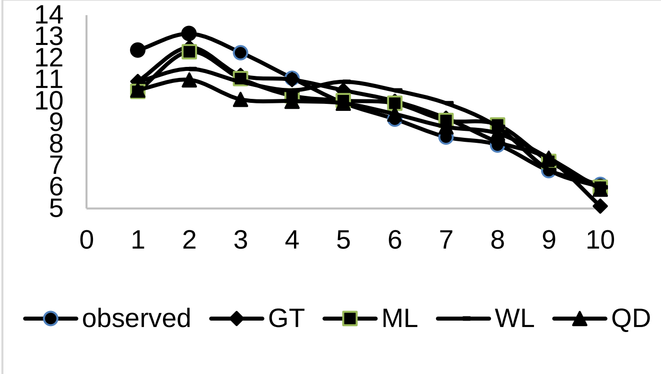

Fig 2: Second lactation milk yield of Frieswal cows as estimated by non-linear models

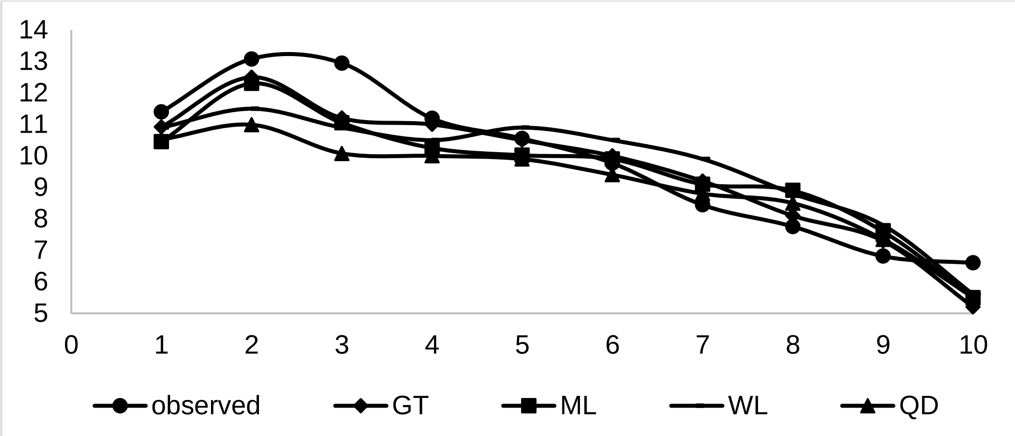

Fig 3: Third lactation milk yield of Frieswal cows as estimated by non-linear models

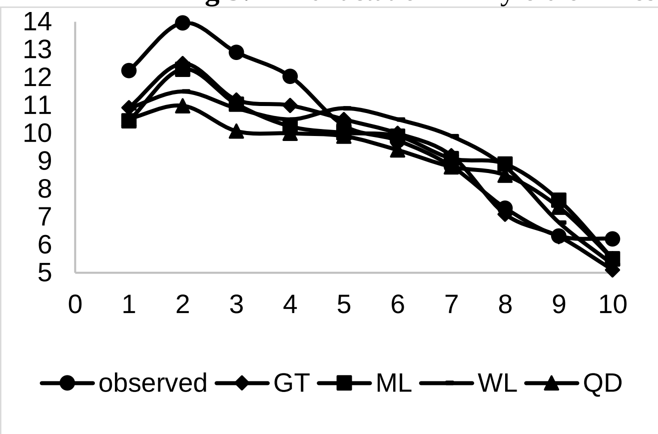

Fig 4: Fourth lactation milk yield of Frieswal cows as estimated by non-linear models

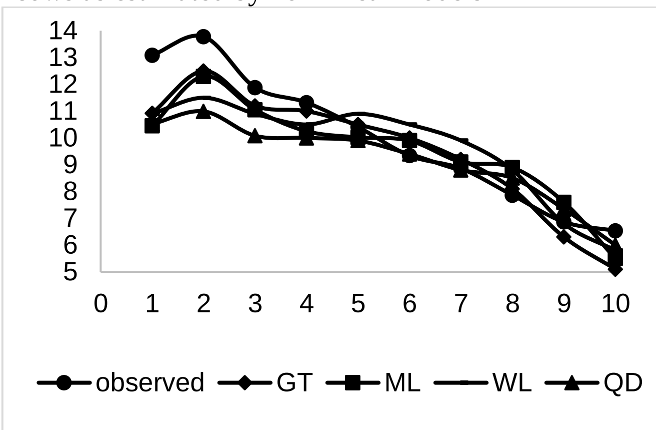

Fig 5: Fifth lactation milk yield of Frieswal cows as estimated by non-linear models

The estimated persistency was different between the evaluated models, which suggests differences in the ability to capture the declining phase of the lactation curve. The Mixed Logarithmic (ML) model yielded the highest persistency estimates (3.30-3.75 months) which indicated its ability to maintain milk yield after peak production and to better describe the ascending and descending phases of lactation. The Gamma-Type (GT) model projected moderate persistency (2.21 to 3.41 months) which is a balanced presentation of the peak yield and the consequent decline. In comparison, the Wilmink (WL) model gave a lower persistency (2.15–2.60 months), suggesting a smoother but slower decrease in milk yield. The QD model had the widest range of persistency (1.18–3.16 months), indicating inconsistent performance across lactations. In general, the ML model was better at predicting sustained milk yield, while the GT model gave a balanced description of the lactation persistence. Dhanoa (1981), Silvestre *et al*. (2006) and Wood (1967) have reported similar observations.

Table 3: Lactation-wise peak yield, persistency, and timing of peak milk production in Frieswal cows

ML			GT			WL			QD		
Peak Yield (Lit)	Persistency (months)	Months in milk at peak yield	Peak yield (Lit)	Persistency (months)	Months in milk at peak yield	Peak Yield (Lit)	Persistency (months)	Months in milk at peak yield	Peak Yield (Lit)	Persistency (months)	Months in milk at peak yield
30	3.75	6.53	18.5	3.41	5.66	9.45	2.6	2.76	10.2	3.16	2.71
27.1	3.49	5.7	16	3.33	5.31	12.1	2.32	1.07	11.3	3.09	1.66
28.1	3.41	5.5	19.6	3.25	5.1	13.7	2.23	2.04	12	3.05	2.14
28.5	3.33	5.31	20	2.76	4.03	15	2.15	1.9	12.7	3.03	2
30.2	3.3	5.21	29.3	2.21	4.02	13.4	2.27	2.39	12.7	1.18	0.12

The month of peak milk yield also differed between models. The ML model predicted the latest peak (5.21–6.53 months) indicating a longer milk production before reaching the maximum yield and supported its higher persistency estimates. The GT model estimated the peak production at mid-lactation (4.03–5.67 months), whereas the WL and QD models predicted the peak production earlier, between 1.07 to 2.76 months and 0.12 to 2.71 months respectively. These results suggest that the ML model is more appropriate to describe extended lactation with sustained production, while the GT model gives an even description of the dynamics of milk yields. In contrast, WL and QD models are more suitable to describe the lactation curves with the early peak and the relatively fast decline. The results agree with previous studies which have shown that biologically based nonlinear models are more accurate in describing lactation patterns of dairy cattle than empirical polynomial models (Wood, 1967; Dhanoa, 1981).

Conclusion

In the present study, it was observed that the nonlinear models were better in describing the lactation curves of Frieswal cows up to fifth lactation. The peak milk yield was generally obtained in the second month of lactation and the fourth lactation had the highest overall milk production. The Mixed Logarithmic (ML) model presented the best fit to the first lactation among the models analyzed. From the second to the fifth lactations, the Gamma-Type (GT) model showed the best predictive ability, as indicated by the higher adjusted R2 and lower RMSE, AIC and BIC values. In contrast, the Quadratic (QD) model showed relatively poorer performance in all lactations. The ML model predicted higher peak yields, greater persistency and later peak, indicating that it can describe lactations with prolonged milk production. Whereas, the lactation curves obtained from the GT model were more balanced and biologically plausible, which makes it more suitable for modeling of mature Frieswal cows. Results indicate that the choice of lactation curve model based on parity can increase the accuracy of milk yield prediction and can facilitate more effective genetic evaluation, breeding programmes and dairy herd management.

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Data Availability

The datasets generated and analyzed during the current study are available from the corresponding author upon reasonable request.

Ethics Approval

The study used archived lactation records of Frieswal cows. The use of data for research purposes was approved by (IAEC) of JNKVV.

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Contribution by Authors

The first and second authors were responsible for study design, result interpretation, and manuscript preparation, while all three authors contributed to data analysis. All authors reviewed and approved the final version..

Conflict of Interests

There is no conflict of interest.

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